PHYSICAL PROBLEMS OF ARTIFICIAL MAGNETOSPHERIC PROPULSION

J.T. Mendonça
GoLP, Instituto Superior Técnico, 1049-001 Lisboa, Portugal
E-mail: titomend@ist.utl.pt

A.L. Brinca
R. Fonseca
J. Loureiro
L.O. Silva and I. Vieira
GoLP, Instituto Superior Técnico

We discuss here some physical problems related to the recently proposed scheme for solar sailing, using an artificial magnetosphere. We will concentrate on the forces acting on the plasma bubble, and their transfer to the spacecraft. Upper and lower limits of the force acting on the bubble are established. The results of test particle dynamics are presented, concerning the interaction of the solar wind with the modified magnetic dipole resulting from the spacecraft coils and the plasma expansion. Propagation of the forces along the magnetic flux tubes, from the bubble magnetopause down to the spacecraft vicinity, is discussed by using a simple MHD theoretical model. Emphasis is made on the distribution of currents flowing in the immediate vicinity of the spacecraft. Finally, results of PIC code simulations of the magnetized plasma expansion are presented and an overall qualitative picture of the physical processes is given, with a discussion of the strategy for obtaining more quantitative estimates of the magnetospheric propulsion efficiency.

Introduction

We discuss here some physical problems related to the Mini-Magnetosphere Plasma Propulsion (M2P2) scheme for solar sailing and outer planet exploration. This scheme was recently proposed by Winglee el al. [1], and assumes that a large plasma bubble is formed and expands around the spacecraft (s/c), taking the size of several kilometers, and is forced to move due to the solar wind pressure. The proposed artificial magnetosphere could be created by a helicon plasma source, and confined by the magnetic dipole generated by a coil, also installed at the s/c. The plasma particles could stay attached to the s/c due to magnetization.

The plasma produced by the plasma source should be dense and warm (n and T of the order of $5 \times 10^{13} \text{ cm}^{-3}$ and 5 eV, so that a discharge of the helicon type fulfills, in principle, these requirements. Most probably, the helicon produces a weekly ionized, collisional gas with $T_e > T_i$ that is injected into the magnetic field generated by the coil.

The success of the mini-magnetospheric plasma propulsion (M2P2) idea hinges on the value of the force that acts upon the spacecraft (S/C). We also discuss here several aspects that might intervene in its estimate. Upper and lower limits of the force acting on the bubble are established. The results of test particle dynamics are presented, concerning the interaction of the solar wind with the modified magnetic dipole resulting from the spacecraft coils and the plasma expansion. Propagation of the forces along the magnetic flux tubes, from the bubble magnetopause down to the spacecraft vicinity, is discussed by using a simple MHD theoretical model. Emphasis is made on the distribution of currents flowing in the immediate vicinity of the spacecraft. Finally, results of PIC code simulations of the magnetized plasma expansion are presented and an overall qualitative picture of the physical processes is given,
with a discussion of the strategy for obtaining more quantitative estimates of the magnetospheric propulsion efficiency.

**Forces Acting on the Bubble**

While analyzing the complex interaction between the solar wind (SW) and the artificial mini-magnetosphere, perhaps the first attempt at estimating the force on the spacecraft comes from the realization that the momentum of the SW particles changes and its conservation requires that it should be picked up by the S/C. However, leaving aside the difficulties associated with a realistic calculation of the variation in the SW particle momentum, one cannot conclude that this change is totally absorbed by the S/C: the momentum of the artificially injected plasma particles is also modified during the interaction with the SW. An alternative way of addressing the calculation of the force on the spacecraft relies on the consideration of the coil in the S/C.

First, we should notice that the gravitational attraction of the Sun at 1 A.U. is of the order of $F_{S/kg} = G \cdot M \cdot m( = 1 \text{ kg}) \cdot (1 \text{ A.U.})^2 = 5.93 \text{ mN/Kg}$. For a s/c prototype with 100 kg this corresponds to $F_S = 0.6 \text{ N}$. This value gives an estimate of the force necessary for the M2P2 method to work.

Let us now make an upper estimate of the force acting on the plasma bubble. For simplicity, we can define the boundary of the plasma bubble created around the spacecraft as the region where the solar wind pressure equals the magnetic pressure of the artificial magnetic dipole. We can call it the magnetopause of the artificial magnetosphere. The number of particles hitting the surface of the magnetopause, per unit time and per unit surface is $N = N_0 \cdot v \cdot \cos(\phi)$, where $N_0$ is the density of solar wind protons (~ 5/cc near the Earth), $v$ is the velocity of the solar wind (~ 400 km/s), and $\phi$ is the angle of incidence. If we assume specular reflection of the protons on the magnetopause, they will suffer a change of momentum equal to $2 \cdot m_p \cdot v \cdot \cos(\phi)$, where $m_p$ is the proton mass. So, the pressure of the solar wind will be $p = 2 \cdot N_0 \cdot m_p \cdot v^2 \cdot \cos^2(\phi)$. We can then calculate the force of the solar wind on the magnetopause, by assuming that this surface is approximately spherical with a radius $R$. The result is:

$$F = \int p \cdot dS = \pi R^2 N_0 m_p v^2$$  \hspace{1cm} (1)

This expression gives the upper limit of the force acting on the plasma bubble, and this result is illustrated in Figure 1.
Let us now look at the force acting on the s/c itself. The spacecraft creates its own magnetic dipole, which will eventually confine the plasma produced by a helicon source. The expected characteristics of the coil are: number of turns: \( n_c = 1000 \), current in the coil: \( I_c = 10 \) A, and coil radius: \( a_c = 10 \) cm. The resulting value for the magnetic dipolar momentum is: \( m_c = \pi a_c^2 n_c I_c = 300 \) A m\(^2\) and, assuming that the axis Oz coincides with the direction of the magnetic dipolar momentum, we can write \( \tilde{m}_c = m_c \hat{e}_z \).

The interaction of the solar wind with the plasma bubble will lead to the formation of induced currents, which will be responsible for the deformation of the magnetic dipolar configuration. The magnetic force acting on the s/c will then be due to the magnetic field \( \vec{B} \) created by these induced currents:

\[
\vec{F} = \nabla (\tilde{m}_c \cdot \vec{B}) = m_c \nabla B_z \tag{2}
\]

Or, assuming that the gradient is directed along the axis Ox:

\[
\vec{F} = m_c \frac{\partial B_z}{\partial x} \hat{e}_x \tag{3}
\]

As a simple model for the currents induced in the plasma in the immediate vicinity of the spacecraft, we consider a linear current \( I \) aligned with the axis Oy, flowing at a distance \( x \) from the spacecraft. The resulting magnetic field at the spacecraft position will be written as: \( \vec{B} = \mu_0 I (2\pi x)^{-1} \hat{e}_z \). The resulting magnetic force acting on the spacecraft will then be determined by

\[
F = m_c \left| \frac{d\vec{B}_z}{dx} \right| = 6 \times \frac{I}{x^2} \ N \tag{4}
\]
This means that, in order to obtain a force similar to attractive force from the Sun, \( F \approx F_S \approx 1 \text{ N} \), we need to have a current \( I > 10^4 \times x^{-2} \text{ A/m}^2 \). For a linear current located at a distance \( x = 1 \text{ m} \) from the s/c, we would need a current larger than ten kilo-Ampere: \( I > 10^4 \text{ A} \). This is an extremely large value of induced currents near the s/c, but the actual plasma currents are induced over a large volume inside the magnetic bubble and are not concentrated as assumed in the present model. The interest of the model is that, whatever the actual volume distribution of currents is, it will have to be responsible for the same value of the magnetic field gradient.

Let us now look at the force propagation along field lines. The solar wind, moving with a velocity \( \tilde{v}_0 = v_0 \hat{e}_x \), perturbs the magnetic flux tubes of the magnetic dipole created by the s/c coils. This perturbation can is described by the MHD equations. These equations relate the velocity of the medium \( \tilde{v} \) with the pressure \( p \), the current density \( \tilde{J} \) and the magnetic field \( \tilde{B} \)

\[
\rho \left( \frac{\partial \tilde{v}}{\partial t} + \tilde{v} \cdot \nabla \tilde{v} \right) = -\nabla p + \tilde{J} \times \tilde{B} \tag{5}
\]

where \( \rho \) is the plasma density, and

\[
\frac{\partial \tilde{B}}{\partial t} = \nabla \times (\tilde{v} \times \tilde{B}) \tag{6}
\]

If we assume a simple geometry such that: \( \tilde{v} = (v_x, 0, 0) \) and \( \tilde{B} = (B_x, 0, B_y) \), and use the constancy of the total plasma pressure: \( p + \mu_0 B_x^2 / 2 = p_0 \) we can obtain the propagation equation for the magnetic field perturbation \( B_x \):

\[
\frac{\partial^2 B_x}{\partial t^2} = v_A^2 \frac{\partial^2 B_x}{\partial z^2} \tag{7}
\]

where \( v_A \) is the Alfven velocity, \( v_A = B_y / \sqrt{\mu_0 \rho} \). This shows that the perturbations induced by the solar wind propagate without significant losses along the magnetic flux tube, with the Alfven velocity, from the magnetopause down to the spacecraft. Particular solutions for this model were discussed long time ago by Scholer [2]. The collisional plasma located in the near vicinity of the spacecraft will eventually introduce some dissipation. But, apart from that, the force is transmitted without attenuation along the magnetic flux tubes.

In order to complete this qualitative discussion let us now use a test particle approach to establish a lower limit for the force produced by the solar wind over an artificial magnetosphere. We assumed that the magnetic field existing inside the plasma bubble is that of a modified magnetic dipole:

\[
\tilde{B} = \tilde{b} K r^{-\alpha} \tag{8}
\]

where \( \alpha \) is the decay factor (the usual magnetic dipole would be \( \alpha = 3 \)), \( K \) is the magnetic field amplitude that depends of the current and dimensions of the coil installed inside the s/c, \( \tilde{b} \) is a unit vector giving the orientation of the magnetic dipole, and \( r \) is the distance of the particle to the spacecraft. A simple simulation code has been developed to solve the test particle trajectory in such a modified magnetic dipole. This allows us to determine the momentum transfer to the particle due to its collision with the magnetic field as a function of
its impact parameter and initial velocity, and thus by momentum conservation, and after averaging over the particle population of the solar wind, to obtain the force acting on the magnetic dipole. Here we neglect the possible collisions with the particles of the plasma bubble, which means that we can only get a lower limit to the force acting on the dipole. The area of simulation is a cube of 30 km x 30 km x 30 km. A uniform flux of protons from the solar wind is introduced along the positive Ox direction. The magnetic dipole is oriented along the Oz direction, $\mathbf{b} = \hat{e}_z$. Both the magnetic field and particle position-velocity are simulated in three dimensions. Using the Runge-Kutta method, the particles are moved during a step of time, named $dt$. The velocity of the particles is updated assuming the presence of the magnetic field created by the modified dipole, which is not considered self-consistent.

Figure 2. Total force versus magnetic decay factor

Several values of the decay factor $\alpha$ (between 3 and 1) were used and the results are shown in Figure 2. We conclude from here that a significant value for the force can only be attained for $\alpha = 1.2$, which means that the plasma expansion mechanism and the subsequent action of the solar wind on the expanded bubble have to be effective. For even smaller values, it is possible to observe a saturation on the force created. This is due to the limited volume of simulation used in our code, and has no physical relevance.

Kinetic Modeling

Previous studies employed MHD models to validate the assumptions of the magnetic field configuration after the mini-magnetosphere formation. However, as also mentioned in the original paper [1], kinetic effects can play an important role in the dynamics of the plasma-magnetic sail unfolding, in particular, associated with instabilities and wave excitation, that might damp some of the kinetic energy release in the plasma, and can give rise to a foamy sail.

Due to the non-trivial magnetic field configuration arising in a mini-magnetosphere,
kinetic simulations are required to understand the plasma sail expansion into a dipole magnetic field configuration. The interaction of the solar wind with this plasma sail also plays a crucial role in the whole mechanism, and needs to be taken into account. It is clear that an electrostatic particle-in-cell simulation cannot model such a system. Only a full electromagnetic PIC code can deal with the currents generated in the plasma that lead to conversion of the plasma kinetic energy into magnetic fields. We have used a modified version of the object-oriented parallel particle-in-cell code OSIRIS [3], in order to include cathodes and externally applied fields. The full problem can be split in two different aspects: (i) plasma expansion into a dipole magnetic field configuration, (ii) interaction of the solar wind with a plasma immersed in a modified magnetic dipole field.

Figure 3. Radial decay of the magnetic field for initial, mid and final times of plasma expansion in the polar plane of the magnetic dipole

Here we show results of 2D PIC code simulations, performed in two complementary versions of the first problem configurations. The simulations were done with a modified version of the code OSIRIS [3]. We have used a mass ratio of $m_p/m_e = 400$, an injection velocity of $3 \times 10^{-2}$ c, thermal velocity of $3 \times 10^{-3}$ c, and we have continuous injection of plasma with density $n_0$ and a Gaussian profile of width $5.0 \, c/\omega_{pe0}$. The $\beta$ parameter (defined as usual by the ratio of the plasma pressure over the magnetic pressure) was taken as 10% at maximum of the B-field. We have assumed a dipole strength equal to $2.0 \, m_e \, c^4/e \, \omega_{pe0}^2$. We have used normalized units, with space normalized to $c/\omega_{pe0}$, time to $1/\omega_{pe0}$, charge to the electron charge $e$, mass to the electron mass $m_e$, and magnetic field to $m_e c \, \omega_{pe0}/e$. The duration of the runs corresponds to $300/\omega_{pe0}$ with time steps $\Delta t = 4.9 \times 10^{-3}/\omega_{pe0}$, and the number of macro-particles was $20 \times 10^6$. The data generated in each run is larger than 125 GB.

The first configuration corresponds to a plasma expansion in the polar plane, or the plane Oxy which contains the two poles of the magnetic field. The magnetic field lines are here parallel to the computational plane. The other configuration corresponds to the equatorial
plane Oxz, perpendicular to the magnetic field lines. The results of the total magnetic field spatial decay, for these two configurations are shown on Figures 3 and 4, for the initial, intermediate and final times of the runs. We can clearly observe the drag of the magnetic field lines due to the plasma expansion, especially in the equatorial plane where a decay of $1/r$ is observed.

![Graph of magnetic field decay](image)

**Figure 4. Radial decay of the magnetic field for initial, mid and final times of plasma expansion in the equatorial plane of the magnetic dipole**

**Conclusions**

Several physical problems relevant to the M2P2 propulsion scheme were discussed here. In particular, upper and lower limits of the force acting on the magnetic bubble were established. The upper limit is based on specular reflection at the artificial magnetopause, considered as a kind of opaque and totally reflecting barrier. The lower limit was based on the dynamic reflection of solar wind particles by the modified magnetic dipole field. We have also briefly mentioned how the perturbed currents, originating near the artificial magnetopause, can propagate down to the vicinity of the spacecraft, with the Alfvén speed and with negligible losses. The resulting magnetic force on the spacecraft is due to large local currents. In order to determine the local current distribution and to establish a convincing value for the magnetic force acting on the spacecraft, we have used PIC code simulations. The first results of these calculations show that the plasma ejected from the spacecraft can indeed drag the magnetic field lines. Expansion of the magnetic field radial decay, from the initial $1/r^3$ down to $1/r$ was observed, thus confirming previous results based on MHD simulations. PIC code simulations of the solar wing interaction with the magnetized plasma bubble will be presented somewhere. A coherent qualitative picture of the process was established. A more quantitative view (of the current distributions) will imply the use of PIC
and hybrid code simulations. Another important aspect of M2P2 is related with the plasma formation, which as also been analyzed by us and will be presented in a future publication.

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References

